An Analysis of Everyday Mathematics in Light of the Third International Mathematics and Science Study

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In the past 20 years, several studies have compared mathematics and science achievement around the world.[1] During the 1995 – 1996 school year, 41 nations took part in the most recent and largest of these studies, the Third International Mathematics and Science Study (TIMSS). Test results from TIMSS confirm and extend finding from the earlier studies (e.g., Stevenson, Lee, & Stigler, 1986; McKnight et al., 1987) and provide important insights into how fourth-, eighth-, and twelfth-grade students in the United States perform in mathematics and science relative to their peers in other nations. Because TIMSS included videotaped lessons, teacher surveys, and analyses of textbooks along with achievement tests, this study provides the most comprehensive picture so far of how curriculum, national goals, and instructional practices affect students' progress in mathematics around the world.

While U.S. fourth graders perform above the international average on the TIMSS mathematics test, our students fall below the average by eighth grade, and are at the bottom by twelfth grade. In fact, the United States was the only country with fourth graders above the international average and eighth graders below. These results have caused educators and researchers to search for the reasons behind the mediocre U.S. performance. TIMSS casts doubt on the validity of certain previously suggested explanations for U.S. students' poor achievement, such as the amount of time spent on math instruction or homework. TIMSS data show, for example, that on average fourth and eighth graders in the United States spend more time in math instruction and have more math homework than students in other nations.

Other aspects of mathematics instruction, such as the nature of the textbook, seem more likely to be the cause of U.S. under-performance. The TIMSS researchers suggest five probable reasons for U.S. under-performance (Frase et al., 1997; Peak, 1996; Schmidt, McKnight, and Raizen, 1996):

- Textbooks in the United States are not as challenging as are those in other nations.
- Mathematics lessons in the United States focus on rote procedures instead of conceptual understanding and problem solving.
- Topics such as geometry, measurement, and algebra are underrepresented in the curriculum.
- The United States curriculum is "a mile wide and an inch deep," lacking a focus at each grade.

[1]For a review, see Robitaille & Travers (1992).

- Although U.S. teachers are aware of reform ideas, they less likely to implement those ideas than their peers abroad.

While state and district goals, the NCTM Standards (National Council of Teachers of Mathematics, 1989, 1991, 1995) and standardized tests all influence how mathematics is taught, many researchers believe that the textbook is the most important single factor in what mathematics is taught and how it is taught (Begle, 1973; National Advisory Committee on Mathematics Education [NACOME], 1975; Suydam, 1979; Fey, 1980; Pollak, 1983; Flanders, 1987; Schoenfeld, 1988; Barr, 1988; Porter, 1989; Slavin, 1990; Romberg, 1992; Davis, 1992; Ball & Cohen, 1996; Olson, 1997). The purpose of this report is to detail how the University of Chicago School Mathematics Project's curriculum *Everyday Mathematics* (EM) has anticipated and addressed the five concerns raised by the TIMSS researchers.

Textbooks in the United States are not as challenging as those in other nations

Even before the Second International Mathematics Study characterized the U.S. curriculum as "underachieving" (McKnight et al., 1987), Max Bell, the Director of the UCSMP Elementary Component, found that textbooks routinely ignore the skills and understandings that students bring to school (Bell and Bell, 1988).

> As we worked with teachers and with children, we became convinced that most children begin school with considerable intuitive knowledge of mathematics. We found that schools and schoolbooks did not take advantage of children's rich store of mathematical understanding. We learned of pioneer programs in the United States and other countries in which children responded enthusiastically to a richer mathematics curriculum. These programs demonstrated that it was feasible to do far more with mathematics that was usual in most elementary schools, provided that the learning experiences were anchored in concrete, real-life experiences, and provided that children were encouraged to construct knowledge and concepts for themselves (University of Chicago School Mathematics Project, 1997: *Fifth Grade Everyday Mathematics Teacher's Manual*, pp. iii – iv).

Although TIMSS and other studies (e.g. Fuson, Stigler, & Bartsch, 1988) suggest that U.S. school mathematics lags behind Japanese and German curricula by a year or more, EM lessons are generally a year or more ahead of topics in other U.S. programs. Where traditional mathematics programs are slow and repetitive, *Everyday Mathematics* picks up the instructional pace, increasing both the depth and the breadth of the mathematics taught. Eight mathematical content strands, reflective of the NCTM standards, are investigated at all grade levels. For example, concepts in algebra begin in kindergarten and first grade through activities such as “What’s My Rule” and the Frames and Arrows routine. By fourth grade, students are solving equations with variables, plotting points on the coordinate grid, and beginning to investigate relationships between equations, tables, and graphs. While traditional elementary books focus on arithmetic through middle school, leaving students ill-prepared for algebra (Flanders, 1987), four of the 10 units in *Sixth Grade Everyday Mathematics* address algebraic topics. Because of this rich experience, UCSMP believes that at least half of the students who complete *Sixth Grade Everyday Mathematics* will be ready for algebra in seventh grade. Rather than underestimating students’ abilities, UCSMP sets the goals high, believing that U.S. students are not only up to the challenge, but relish doing more interesting mathematics.

Although it is a more ambitious curriculum, *Everyday Mathematics* is designed for *all* students, not only the most mathematically gifted. Topics are chosen so students can build on their knowledge and intuitions about mathematics and the world about them. Hands-on activities, manipulatives, mathematical tools, and multiple representations for mathematical concepts provide support for students as they progress from intuitive to formal thinking. Small-group work, games, and whole-class discussions also help students of different abilities learn from each other. Finally, *Everyday Mathematics* has many games and activities that emphasize everyday uses of mathematics, stimulate interest, and motivate learning.

Achievement studies show that students can handle the challenging content of EM. A longitudinal study conducted by Dr. Karen Fuson of Northwestern University has found that first grade EM students scored much higher than traditionally taught U.S. students, and perform more like their peers in Japan and Taiwan (Drueck, Fuson, Carroll, and Bell, 1995). In second grade, EM students scored higher than both U.S. and Japanese students on a test of number sense. Evaluations conducted by UCSMP provide further evidence for higher achievement of students in the EM curriculum (Carroll, 1996a; Carroll, 1998). (See Everyday Learning Corporation (1996, 1998) for further information about EM student achievement.)

TIMSS finding: Mathematics lessons in the United States focus on rote procedures instead of conceptual understanding and problem solving

The TIMSS study reveals striking differences in how U.S. and Japanese teachers plan and deliver lessons. When asked about the goal of a lessons, eighth-grade teachers in the United States cited acquisition of a procedural skill 60% of the time; Japanese teachers cited skills and procedures only 27% of the time (Peak, 1996). Videotaped lessons confirm that Japanese teachers were more likely to present lessons that emphasized problem solving and the development of conceptual understanding, while U.S. teachers were more likely to structure lessons around practicing a rote procedure. Table 1 highlights the differences in mathematical lessons in these two nations.

Table 1: Typical lesson structures in Japanese and U.S. middle-school mathematics classes

Typical lesson in Japanese classroom	Typical lesson in U.S. classroom
• Teacher poses a problem • Students struggle with the problem. • Various students present ideas or solutions to the class. • Class discusses the class's solutions. • Students practice similar problems.	• Teacher instructs students in a concept or skill • Teacher solves example problems with class. • Students practice on their own while the teacher assists individual students.

Note: Table is from Pursuing Excellence (Peak, 1996).

Lessons in the EM curriculum are similar to Japanese lessons, focusing on student's mathematical solutions and the examination of alternative strategies. In fact, the first column in Table 1 might be used to describe a typical EM lesson. While skills and procedures are important in *Everyday Mathematics*, students are generally encouraged to develop, use, and discuss their own methods for solving problems first. This problem-solving approach is used for several reasons. Students' methods are generally more meaningful to them, and often as powerful as standard algorithms. The standard school algorithms are not always the best method for solving a problem. Furthermore, as students develop and discuss their own methods, they develop better conceptual understanding and problem-solving skills (University of Chicago School Mathematics Project, 1998):

> *Everyday Mathematics* believes that children should be encouraged to invent and share their own ways of doing operations, rather than having everyone learn the same standard algorithm at the same time. Learning algorithms too early may actually inhibit the development of children's mathematical processes and cause them to miss out on the rich experiences that

> come from developing their own computational strategies. (*Everyday Mathematics Teacher's Reference Manual,* Grades K – 3, p. 93).

This is not to say that skill mastery is not expected in *Everyday Mathematics*. Fact automaticity is expected and nurtured through various games and practice routines. As for paper-and-pencil algorithms, EM emphasizes "low-stress" algorithms, such as lattice multiplication, that are easier to learn than traditional procedures and are often easier to understand as well. Although these procedures are practiced in MathBoxes and other routines, lessons rarely focus exclusively on the acquisition of a rote procedure. Instead, students apply their procedures to real-life problems. During many problem-solving activities, students may choose between an invented procedure, a pencil-and-paper method (including the standard algorithms), or a calculator.

Research shows that this approach works. While in *Everyday Mathematics* more time is spent solving problems than practicing rote procedures, students do as well on symbolic computation and much better on word problems. Research also shows that EM students develop much stronger number sense and better metacognitive control about which procedure to apply than do students in a computation-oriented curriculum (Carroll, 1996b).

TIMSS finding: Topics such as geometry, measurement, and algebra are underrepresented in the curriculum.

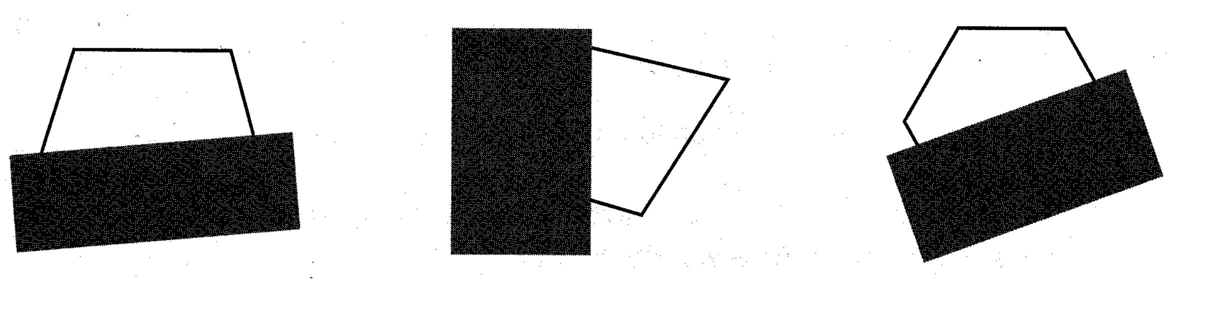
While their overall test score is above the international average, U.S. fourth-graders score below average in Measurement, Estimation, and Number Sense. By eighth-grade, U.S. students are significantly below the international average in the both the Geometry and Measurement content strands.[2] The National Assessment of Educational Progress (NAEP) provides further evidence for the relative lack of progress in geometry and measurement. NAEP results (Lindquist & Kouba, 1989) show that although eighth-grade students can name and identify common geometric figures, they have difficulty answering questions which require reasoning about properties and relationships. However, reasoning about properties and relationships is at the core of mathematics, and these deeper understandings are a prerequisite for success in proof-oriented geometry courses. Traditional

[2] By eight-grade, U.S. students scored above only five other nations in Measurement and six other nations in Geometry.

textbooks give little attention to these topics in elementary and middle school, where computational topics dominate.

In contrast, geometry and measurement are investigated in greater depth at each grade level in *Everyday Mathematics*. For example, Figure 1 shows a typical assessment question from *Fourth Grade Everyday Mathematics*. Besides the use of more sophisticated geometric language (e.g., *parallelogram* and *polygon*), students must reason about the properties of these figures and use problem-solving skills such as process-of-elimination to identify the figures. In this way, students go beyond the rudimentary identification of shapes typical of elementary curricula towards analysis of geometric properties and relationships. Geometric topics included in fourth-, fifth-, and sixth-grade EM include analysis of plane and solid figures, constructions with compass and straightedge, transformations and symmetry, angular measurement, and area and perimeter. Geometric topics are also linked to other topics such as fractions, computation, and measurement. Along with algebra, geometry is a major focus of the EM 4-6 curriculum.

1. Part of each polygon below is hidden. One of the 3 polygons is a **parallelogram**, another is a **trapezoid**, and another is a regular **hexagon**. Write the correct name of each polygon on the line.



A geometry question from *Fourth Grade Everyday Mathematics*

The Measurement strand in EM is similarly ambitious. Beginning in kindergarten, students use tape measures, rulers, templates, and other tools to measure objects in both metric and customary units. In addition to direct measurement, students are asked to estimate and measure length, distance, area, volume, and weight. "Personal references" for common units of measurement are developed to help in estimation and in developing a better understanding of measurement. For example, the width of a finger might be used as a centimeter reference.

Research shows that this emphasis on geometry and measurement yields big gains in students' readiness for more abstract concepts. Compared to students in a more traditional curriculum, EM students score substantially higher on tests of geometric reasoning and conceptual understanding. In fact, 29% of EM sixth graders appear ready for a more proof-oriented geometry course, in contrast to 7% of their peers in a traditional program (Carroll, 1998).[3] Because of the design of the EM curriculum, students make continual progress in geometry, measurement, and other topics, from kindergarten through sixth grade.

TIMSS finding: The United States curriculum is "a mile wide and an inch deep," lacking a focus at each grade.

The discussion above indicates that insufficient progress has been made in reshaping mathematics education in the United States since the Second International Mathematics Study described the U.S. mathematics program as an "underachieving curriculum" (McKnight, Crosswhite, Dossey, Kifer, Swafford, Travers, & Cooney, 1987). According to TIMSS researchers, mathematics textbooks in the United States lack focus and depth. Several reasons have been suggested for this "splintered vision" of mathematics education (Schmidt, McKnight, and Raizen, 1996). First, in order to maximize salability, many U.S. publishers include a potpourri of topics that will match the goals of many different states and districts. Unfortunately, the resulting overstuffed curriculum treats most topics superficially. Second, reforms and new topics called for by NCTM and other groups have been added to textbooks without substantial deletions of outdated material, again encouraging overcrowding the curriculum. Third, tests of various sorts, often with divergent goals, continue to guide what is taught. Thus, many teachers must prepare students for standardized tests as well as for state assessments that are better aligned with the *Standards*. For these reasons, many U.S. texts are bloated, lacking a coherent vision of the mathematics that is important to learn (U.S. Department of Education, 1997).

Because of how it was developed, *Everyday Mathematics* is a more coherent program. While other curricula are "a mile wide and an inch deep," repeating topics year after year, *Everyday*

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[3] These analyses were based on the van Hiele model for geometric understanding.

Mathematics has developed an ambitious spiraling curriculum in which students investigate mathematical ideas in greater depth each year. While each content strand is revisited each year in EM, the focus shifts from number and number sense in the primary grades to algebra and geometry in the middle grades. Several factors made this careful ever-deepening treatment of key topics possible.

First, the EM curriculum was developed sequentially, one grade at a time, beginning with kindergarten. The development cycle from the field testing of kindergarten EM through the commercial publication of sixth-grade EM took approximately 10 years. With this grade-by-grade development, the EM author team was able to develop the mathematics coherently, building each year on the previous year. The extended development period has also allowed the authors to incorporate findings from their own ongoing studies of students using the curriculum. And because the university-based author team is relatively small, topics can be more easily coordinated with each other. This contrasts sharply with the traditional production of textbook series, where multiple grades are produced simultaneously by different teams of writers, often with little communication across grades.

Second, *Everyday Mathematics* incorporates the best curricular practices from the U.S. and abroad. For example, the use of brief and frequent practice in Soviet elementary texts has inspired MathBoxes and similar routines in EM. Since its inception in 1983, UCSMP has been a leader in bringing to the attention of U.S. mathematics educators the best ideas in mathematics education from around the world (Wirszup & Streit, 1987, 1990, 1992). Moreover, because of its non-commercial funding from the National Science Foundation, Amoco, GTE, and other sources, the EM development team is less market-driven and thus more able to develop a program that includes reform ideas and de-emphasizes topics that are less important in our technological age. Thus, for example, topics like long division are given less emphasis.

Finally, development of *Everyday Mathematics* has also been guided by a coherent vision of the mathematics that students will need in the 21st Century. Bell’s (1974) early proposal of what “Everyman” needs from school mathematics and research by Usiskin and Bell (1983) into the uses and categories of numbers in everyday life have provided an intellectual framework for the development of EM.

In summary, while more traditional programs lack a coherent vision, the EM curriculum has been developed around several principles, including the following:

- Children begin school with a great deal of knowledge and intuition on which to build and are capable of handling a more challenging mathematics program.
- The curriculum should begin with children's experience and work to connect that experience with the discipline of mathematics.
- The mathematical experiences that children have in school should reflect the mathematics that they will need in a highly technological age.
- The mathematics that is investigated at each year should build directly upon experience previous years, with students studying topics in increasing depth.
- The curriculum should include practical routines to help build arithmetic skills and quick responses that are essential in a problem-rich environment.

TIMSS finding: Although U.S. teachers are aware of reform ideas, they less likely to implement those ideas than their peers abroad.

TIMSS results indicate that although U.S. teachers are aware of the NCTM Standards and other reform ideas, they are less likely to implement those ideas in their classrooms than teachers in high-achieving foreign countries. Because of their experience in schools and with previous curriculum efforts, the EM authors are well aware of the difficulties that U.S. teacher face (Bell, 1993):

> Reforms often fail because they do not take into consideration the working lives of teachers. The new curriculum should be practical and manageable, and it should include suggestions and procedures that make teachers' lives easier, at least in the long run. (The University of Chicago School Mathematics Project Newsletter: Autumn 1993, p. 7).

Throughout its development, the EM program has attempted to achieve this by giving teachers a central role at each stage. Classroom teachers from a variety of schools work with the development team in the planning, writing, field testing, and revising lessons and activities. The result is a program that takes into account both the considerable normal pressures on teachers and the even greater difficulties encountered in implementing change.

While changing how mathematics is taught is hard, research by Northwestern University's Karen Fuson shows that EM teachers are very successful in implementing ideas from the NCTM Standards. In classroom observations of second- and third-grade teachers, Fuson found that EM teachers were more likely than either U.S. or Japanese teachers to ask problem-solving and conceptual questions. These questions occurred in nearly every EM lesson observed. Furthermore, the EM teachers consistently asked students to explain their own methods and mathematical thinking. As opposed to more traditional classes, more of an emphasis in EM classes was on student thinking than on the development of rote procedures.

Conclusion

The recent Third International Mathematics and Science Study shows that U.S. students continue to lag behind their peers in other countries. Researchers have suggested several reasons for this, most importantly an unchallenging and unfocused middle-school curriculum. Because of its research base[4], its international perspective, and its unique approach to curriculum development, UCSMP's *Everyday Mathematics* differs substantially from other programs and has anticipated many of the concerns raised by the TIMSS report. In contrast to more traditional programs, in *Everyday Mathematics* students investigate mathematical topics in greater depth each year. As the curriculum moves from the primary grades, the emphasis shifts from number and number sense to algebra, geometry, and data, with the goal that approximately half of the students who complete the program will be ready for algebra by seventh grade. Results from a variety of studies of *Everyday Mathematics* show that U.S. students are up to the challenge of a more demanding mathematics curriculum.

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[4] While some of the research is cited here, a fuller description can be found in "A research-based curriculum: The research foundations of the UCSMP *Everyday Mathematics* curriculum" (Isaacs, Carroll, & Bell, 1997) available from Everyday Learning Corporation.

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