A Research-Based Curriculum:

The Research Basis of the UCSMP Everyday Mathematics Curriculum

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UCSMP Elementary Component

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The Research Basis for *Everyday Mathematics*

During the 1980s, a consensus emerged among mathematics educators about how best to teach mathematics to children in school. The NCTM *Standards* (1989) expressed that consensus and communicated it to a broader audience. *Everyday Mathematics* is based largely on the same body of research that led to the *Standards* consensus. In this paper we describe the research findings that were most influential in the original development of *Everyday Mathematics*.[1]

Infants' and Young Children's Mathematical Knowledge

Before 1980 many people believed infants to be fairly passive and unaware of the world around them. Thanks to the work of Starkey and Cooper (1980), Strauss and Curtis (1981), and others, a very different view emerged, one that showed infants to be active, alert to the world, and aware of differences in numbers of objects and certain additive or subtractive relationships. That pioneering work has been expanded in recent years (e.g., Wynn, 1992) and shows the crucial importance of early experience in building cognitive abilities.

Research in the late 1970s and early 1980s also revealed that preschool children have much richer and more active mathematical minds than had been suspected (Gelman and Gallistel, 1978; Gelman, 1982; Resnick, 1983; Fuson & Hall, 1983; Gelman, Meck, & Merkin, 1986). They found that young children are capable of absorbing a great deal of new material, sometimes more rapidly than adults—for example, in the learning of language—but that neurological windows for learning appear to close at certain points in the development of the brain, making later learning of concepts more difficult. Also during the 1980s, certain supposed constraints on what and when children could learn, hypothesized by Piaget and others, were shown to be artifacts of the research tasks and not truly indicative of the capabilities of children (Walsh, 1991).

[1] *Everyday Mathematics* was developed over a period of more than ten years beginning in about 1985, a time frame that is reflected in the dates of the references in this paper. More recent research has confirmed and extended the findings discussed here. The full text of this paper, including the references, can be found at http://everydaymath.uchicago.edu/about/research-results/.

Many studies confirmed that young children, regardless of social-economic background, possessed considerable informal mathematical knowledge, which the curricula in use at the time failed to use (Riley, Greeno, & Heller, 1983; Carpenter & Moser, 1984; Hiebert, 1984; Cobb, 1985; Baroody & Ginsburg, 1986; Bell & Bell, 1988; Resnick, Lesgold, & Bill, 1990; Carpenter, Ansell, Franke, Fennema, & Weisbeck, 1993). For example, even without instruction, most kindergarten children are capable of solving a wide range of simple addition and subtraction story problems by their own methods (Riley, Greeno, & Heller, 1983; Carpenter and Moser, 1984). Multiplication, division, and fraction problems are also within their reach when manipulatives are available (Carpenter, Ansell, Franke, Fennema, & Weisbeck, 1993). In brief, children know much about addition and subtraction and other operations before formal instruction begins. Research on children's informal solution methods revealed a typical developmental progression from simple counting of objects, to use of more sophisticated counting strategies and relationships, to derived fact strategies, to use of arithmetic facts and number relationships (Bergeron & Herscovics, 1990; Fuson, 1992).

In extensive interviews with children, Max Bell and Jean Bell, developers of *Everyday Mathematics,* showed that knowledge of counting, reading and writing numbers, and problem solving among kindergarten children far exceeded the expectations of kindergarten programs of the time (Bell & Bell, 1988). For example, while most programs spent the entire year focused on counting to 10 or 20, 84% of children *beginning* kindergarten could already count 13 dots, nearly half could count beyond 30, and many could count to 100, read and write numerals such as "57" or "100", act out "equal sharing" division problems, and name common geometric objects.

Typical first grade textbooks of the time expected children to count only to 100 and to read and write numbers to 20, but interviews revealed that many beginning first graders already had whole-number capabilities beyond those and could actually solve simple fraction problems. Even children's "errors" showed substantially greater capabilities than had been supposed. For example, many beginning first graders wrote "four hundred ninety-eight" from dictation as "40098" or "4098" and read the numeral "5004" as "five hundred and four"—wrong answers to be sure, but indicative of readiness for work well beyond the content offered in standard textbooks of that time.

Algorithms

Researchers working in the 1970s and 1980s showed that U.S. children often learn standard computational algorithms with very little understanding (Brown & Burton, 1978; Van Lehn 1983, 1986). Other researchers found that the traditional approach to teaching computation engenders beliefs about mathematics that impede further learning (Hiebert, 1984; Cobb, 1985; Baroody & Ginsburg, 1986).

On the other hand, Kamii and others demonstrated that students are capable of inventing their own effective and meaningful methods for computation (Kamii, 1985; Madell, 1985; Kamii & Joseph, 1988; Cobb & Merkel, 1989; Resnick, Lesgold, & Bill, 1990; Carpenter, Fennema, & Franke, 1992). Furthermore, these experiences were found to improve understanding of place value and enhance estimation and mental computation skills.[2]

Learning in a Social Context

During the 1970s and 1980s, the work of Lev Vygotsky (1962) began to be more widely known in the United States. Vygotsky's research promoted a view of learning based on both individual and social construction, and showed the importance of social functions in supporting and extending learning. Language, tools, and social interactions all assist children in acquiring skills and concepts. For example, a problem that seems beyond the capabilities of a child working alone with paper and pencil can often be solved when appropriate manipulatives are available. When children interact with each other or with adults, then their learning potential, what Vygotsky called the "zone of proximal development," is extended, increasing both the types of tasks that can be accomplished and the amount of learning that takes place. Early learning appears to be greatly enhanced by ongoing interactions between children and their world, including adults in that world. Talking about ideas, with informal error corrections by adults and peers, is often as important as thinking about ideas, and conversations can gradually become internal dialogues that guide the child's progress through a problem.

[2] For a discussion of the *Everyday Mathematics* approach to algorithms, see http://everydaymath.uchicago.edu/teaching-topics/computation/.

Interviews conducted by Bell and Bell (1988) confirmed the relationships between developing mathematical capabilities and social situations. Questions that seemed beyond primary children's capabilities were often solved during the interview. Children would say, "I didn't know I could do that!" The availability of tools further extended what a child could explain, and a simple rephrasing of a question often made it quite understandable. For example, children who could not make sense of the symbol "1/2" or of the expression "12 divided by 3" could easily respond correctly to the request, "Please give me half of these blocks" or "Share these blocks among you, me, and my friend [a doll]." While learning and understanding are sometimes individual activities, they are often social activities, greatly influenced by the situation, the language used, and the materials available.

The Johnson brothers also began to publish their pioneering work on cooperative learning during the 1970s (Johnson & Johnson, 1974, 1978).

The Underachieving Curriculum

In the early 1980s, the UCSMP Resource Development Component began studying mathematics education in the Soviet Union, Japan, China, and other high-achieving countries (Wirszup & Streit, 1987, 1990, 1992). Wirszup found that other nations were much more ambitious in the scope and sequence of mathematics covered. Even in arithmetic, textbooks in other countries presented topics earlier, had a consistent pattern of spaced practice with mixed operations, and included both more types of word problems and more challenging problems than U.S. textbooks (Stigler, Fuson, Ham, & Kim, 1986; Fuson, Stigler, & Bartsch, 1988). For example, although kindergarten and first grade children had notions of doubles and other multiples, a sure grasp of the demands of equal sharing, and a clear understanding of "half of," multiplication and division were not in the U.S. curriculum until late in second or third grade, and then primarily as rote memorization of the simplest facts. Children also had substantial capabilities from their everyday experience with decimals (money), numbers less than zero (winter temperatures), measurement, and geometry. In teaching experiments by UCSMP researchers, children showed readiness for algebra, functions, and data analysis, but all these topics were deferred to later grades or given scant attention in U.S. textbooks of the 1980s. Not surprisingly, in international studies, U.S. students consistently ranked near the bottom in comparisons with their peers in other industrialized nations. (Stevenson, Lee, & Stigler, 1986; McKnight et al., 1987).

Classroom observers found that teaching practices in the higher-achieving nations differ greatly from those in the U.S. For example, researchers found that Japanese elementary teachers employ more child-centered, problem-solving approaches to instruction in mathematics (Stevenson & Stigler, 1992; Stigler & Perry, 1988). Problems are posed in realistic contexts, and students find their own solution methods. To support these explorations, each Japanese student has a tool kit of manipulatives. Following an exploratory lesson segment, the Japanese teacher asks students to explain their reasoning and multiple solutions. This pattern—problem posing, exploration with manipulatives, and discussion of multiple solutions—fits very well with what we now know about how children learn.

The use of mathematical modeling, aided by increasingly powerful computers, had transformed research and practice in many areas. Important decisions in work and daily life required greater knowledge of mathematics, as well as greater problem-solving and reasoning skills—but results from the second NAEP (Carpenter et al., 1981) showed that most U.S. students completed school without basic problem-solving and reasoning skills and with little appreciation of the utility of mathematics. Educators, leaders of industry, and governmental agencies realized that the U.S. was failing to produce citizens competent in the mathematics that would be needed to compete in the twenty-first century (Education Commission of the States Task Force on Education for Economic Growth, 1983; National Commission on Excellence in Education, 1983).

Mathematical Modeling and Problem Solving

Investigations in problem solving (Polya, 1948, 1962; Lesh, Post, & Behr, 1987; Schoenfeld, 1987; Janvier, 1987) showed that an important step in solving a problem is choosing a model or representation for the problem situation. Researchers and theorists stressed the importance of natural language, concrete models, physical or mental visual images (including pictures, graphs, and diagrams), and symbols in representing mathematical ideas (Bruner, 1964a, 1964b; Lesh, Post, & Behr, 1987; Silver, 1987; Hiebert, 1988). Facility with multiple representations, especially the ability to translate among representations, was found to be important in problem solving.

Researchers also noted that the symbolic manipulations that students carry out in school are often disconnected from reality and common sense (Hiebert, 1984, 1988; Baroody & Ginsburg, 1986; Van Lehn, 1986; Silver, 1986; Resnick, 1987b; Kaput, 1987a, 1987b; Romberg & Tufte, 1987). As a result, students produce nonsense but don't realize it, as Van Lehn and others have shown in their analyses of arithmetic errors. Research also showed, however, that if symbolism is closely related to actions and referents that are familiar to young students, then they are able to deal effectively with it (Hiebert, 1984, 1988; Carpenter, Fennema, & Franke, 1992).

Calls for increased tool use in schools were common before 1990. Both research findings (Suydam, 1984, 1986) and theoretical considerations (Bruner, 1964a, 1964b; Hiebert, 1984, 1988; Lesh, Post, & Behr, 1987; Resnick, 1987b) supported increased use of tools (a.k.a. manipulatives) in school. One particular tool coming into use during this period was the hand-held calculator. Bell (1976) recognized that calculators should play a role in curriculum and learning. Initial research (Hembree & Dessart,1986, 1992; Suydam, 1982, 1985, 1987), since confirmed (Smith, 1997), found that calculators can be valuable tools in school mathematics.

Applications and a Broader Curriculum

Interest in using applications in school mathematics increased during the 1970s and 1980s (Sharron, 1979). Bell (1972) made it clear that people's ordinary lives provide a rich source of brief but interesting problems for school arithmetic. Usiskin and Bell (1983) proposed a scheme for categorizing the uses of numbers and operations with numbers, so that the actual uses of numbers could easily be included in an organized way in school mathematics programs.

Bell (1974) outlined content for a new and ambitious mathematics curriculum. In contrast to traditional K-6 textbook programs, the proposed curriculum framework included investigations in measurement, geometry, algebra, and statistics, as well as in arithmetic. Bell's ideas were taken up in a series of authoritative reports on the content of school mathematics (NCSM 1977, 1988; Pollak, 1983; NCTM, 1989).

Pacing and Practice

While research in reading showed that students achieved best when topics were presented at a brisk pace (Barr, Dreeben, & Wiratchai, 1983), most mathematics texts of the 1970s and 1980s moved quite slowly. An investigation by UCSMP of U.S. mathematics textbooks found that from first through eighth grade, more than half of each year's program was typically devoted to a review of topics from previous years (Flanders, 1987). In those textbooks, a topic was typically introduced and practiced for several weeks and then largely ignored until the following year, when it was reviewed, practiced, and perhaps slightly extended. This cycle of annual repetition with little substantive development was severely criticized by researchers who studied U.S. and foreign textbooks (McKnight et al., 1987; Schmidt, McKnight, & Raizen, 1997). Texts that were essentially medleys of disconnected topics arranged in a flat "spiral" were identified as a prime reason for U.S. students' poor performance on international tests.[3]

Besides a brisk pace, research findings from before 1990 supported continuous review and distributed practice. Practice has long been recognized as essential if children are to retain what they learn (Brownell, 1935, 1956; Brownell & Chazal, 1935; Rathmell, 1978; Chase & Chi, 1981; Cook & Dossey, 1982; Coburn, 1989). The positive effects of "spaced" rather than "massed" practice were recognized as early as 1885 when the German psychologist Hermann Ebbinghaus published his seminal work on memory. Over the past century, Ebbinghaus's findings have been repeatedly confirmed and extended (Caple, 1996). Research about the role of distributed, or spaced, practice in the learning of mathematics was summarized in Suydam's 1985 ERIC digest (ED 260891): "Long-term retention is best served if assignments on a particular skill are spread out in time rather than concentrated within a short interval." Transfer of a skill or concept is also more likely to occur when it is practiced in a variety of contexts and situations (Anderson, Reder, & Simon, 1996).

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[3] Note that these findings do not imply that all spiral curricula are necessarily flawed, only that the traditional U.S. ones are. Indeed, Thomas Romberg, the general editor of the NCTM *Standards*, wrote as his first "principle of curriculum engineering" that "The main generic schemata (i.e. measurement, mappings, proportionality) that we wish to develop in school children must be identified and a spiral curriculum built around those conceptual strands" (Romberg & Tufte, 1987).

Meaning and Skill

One of the perennial arguments in education is between those who want students to develop skill in carrying out procedures and those who want students to understand why those procedures work. Like most such either-or dichotomies, however, this is a false choice. In reality, children with weak conceptual understandings are hindered in their skill development, and children with weak skills are handicapped as they work towards higher levels of conceptual understanding (Carpenter, 1986).

Educators have long recognized that concepts and skills develop best when proper attention is given to both. In 1902, for example, Dewey stressed both that learning must be meaningful for the students and that learning must lead students into established disciplines of study. Years later, Brownell pointed out the necessity for a balance between skills and meaning: “In objecting to the emphasis on drill prevalent not so long ago, we may have failed to point out that practice for proficiency in skills has its place too” (1956). More recent researchers have also pointed out the unfortunate outcomes when a proper balance between meaning and skill is not maintained (Skemp, 1978; Baroody & Ginsburg, 1986; Resnick, 1987b).

Staff Development

During the New Math era, scant attention was paid to the staff development needs of elementary school teachers.[4] Part of the Back-to-Basics movement of the 1970s was actually an emphasis on “teacher-proof” materials. Thus, when UCSMP was founded in 1983, the project’s Elementary Teacher Development Component was breaking new ground. One key finding from work carried out in the 1980s by Paul Sally and Sheila Sconiers at UCSMP was that staff development needed to focus on building teachers’ understanding of mathematics. Other work at the University of Chicago showed that while teachers used a variety of teaching formats in areas such as language arts and social studies, including student projects and small-group work, in mathematics instruction by those same teachers was dominated by individuals filling in answers on page after page of arithmetic problems (Stodolsky, 1988).

Development Principles for *Everyday Mathematics*

The research described above and their own experience led the *Everyday Mathematics* authors to a number of principles for curriculum development. These include:

- Children begin school with a great deal of knowledge and intuition on which to build; by making use of this knowledge, far more can be accomplished in the primary grades than has traditionally been supposed.
- The curriculum should begin with children’s experience and should work to connect that experience with the discipline of mathematics; the materials should encourage the children’s own construction of knowledge.
- Curriculum development should proceed grade by grade starting at Kindergarten so that each grade can build on proven outcomes of the previous grade.
- The curriculum should be more than just arithmetic; geometry, data analysis, measurement, probability, algebra, and problem solving can be taught in elementary school; the curriculum should include rich problems, mathematical modeling, and cross-curricular connections.
- The curriculum should be balanced: concepts, skills, facts, and tools are all necessary.
- Excellent instruction is important.
- Reforms must take account of the working lives of teachers; teachers should be active collaborators in designing the curriculum.
- The pace should be brisk.
- Topics should be arranged in a helix; practice should be distributed rather than massed.
- The curriculum should make use of manipulatives, including calculators.
- The curriculum should include practical routines to help build the arithmetic skills and quick responses that are essential in a problem-rich environment.

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[4] High school mathematics teachers did have many opportunities for staff development, but elementary school teachers were largely neglected.

The *Everyday Mathematics* Writing Process: Research-Guided Revision

The principles above guided the initial drafting of the *Everyday Mathematics* materials, which began with Kindergarten in the mid-1980s. The draft materials were written and field-tested one grade at a time. During the field tests, UCSMP staff gathered detailed critiques of the materials from the field test teachers and also carried out rigorous formative evaluation studies (Carroll, 1996a, 1995b, Carroll & Porter, 1994; Hedges, Stodolsky, & Mathison, 1987). The draft materials were revised on the basis of the evaluation studies and teacher feedback, and were commercially published. Finally, extensive summative evaluations of the published materials were carried out by UCSMP and by a group at Northwestern University, often focusing on issues identified as problematic during the field test (Arron, 1993; Carroll, 2000a, 2000b, 1999, 1998a, 1998b, 1997, 1996b, 1996c, 1996d, 1995a, 1993; Carroll & Fuson, 1998a, 1998b; Carroll, Fuson, & Diamond, 2000; Carroll & Isaacs, in press; Carroll & Porter, 1998, 1997; Ding, 1997; Drueck, 1996; Drueck, Fuson, Carroll, & Bell, 1995; Fraivillig, 2001, 1996; Fraivillig, Murphy, & Fuson, 1999; Fuson, 1997; Fuson & Carroll, 1998; Fuson, Carroll, & Drueck, 2000; Fuson, Carroll, & Landis, 1996; Murphy, 1998). Because of this elaborate development process, the production of the first edition of EM took more than 10 years.

This writing process — research-based drafting, field testing with formative evaluation studies, revision leading to commercial publication, and summative evaluation of the final materials — was designed to bridge the gap between research and practice. The overriding goal was to produce practical materials that ordinary teachers could use to significantly improve the mathematics education of their students.

Almost immediately after the publication of *Sixth Grade Everyday Mathematics* in 1996, work began on a second edition. Due to the exigencies of commercial publication, the second edition development schedule was more compressed than the first edition schedule, but a similar writing process was used. As with the first edition, the second edition authors sought detailed feedback from teachers, tried out materials in classrooms, and revised the draft materials accordingly. Results from the Northwestern University longitudinal study, including a series of unpublished reports with recommendations for revisions, were especially useful in the second edition work.

Everyday Mathematics Student Achievement

A large number of studies of *Everyday Mathematics* student achievement have been conducted. These studies have been carried out by four principal groups: (i) the elementary and evaluation components of UCSMP (see above for cites), (ii) an NSF-funded group at Northwestern University, which carried out a five-year longitudinal study of the curriculum (see above for cites), (iii) individual schools and districts using the curriculum (Everyday Learning, 2001, 1998, 1996; Greene, 1996; Briars & Resnick, 2000; Mathematics Evaluation Committee of the Hopewell Valley Regional School District, 1997), and, increasingly, (iv) independent researchers (Hawkes, Kimmelman, & Kroeze, 1997; Woodward & Baxter, 1997; Riordan & Noyce, in press). These studies, which have used a wide range of instruments and methods to measure students' progress and understanding, provide a broad perspective on the effects of the curriculum.

Generally, results indicate the following:

- On traditional topics, such as fact knowledge and paper-and pencil computation, *Everyday Mathematics* students perform as well as students in more traditional basal programs. However, *Everyday Mathematics* students use a greater variety of computation methods and are especially strong on mental computation.
- On topics that have been underrepresented in the elementary curriculum, such as geometry, measurement, algebra, problem solving, reasoning, and communication, *Everyday Mathematics* students score substantially higher that students in more traditional programs. Total mathematics achievement typically increases significantly following the adoption of the curriculum.

The high level of *Everyday Mathematics* student achievement is evidence for the validity of the research on which the program is based and for the robustness of the writing process that produced the finished materials. *Everyday Mathematics* shows that a research-based, *Standards*-aligned curriculum can lead to higher student achievement. The curriculum's wide-scale implementation — approximately 2,000,000 students currently use the materials — demonstrates, moreover, that such a curriculum can succeed in the marketplace.

Conclusion

Like the NCTM *Standards* (1989, 1991, 1995, 2000), the *Everyday Mathematics* curriculum has been influenced by a rich body of research about children learning mathematics. Many sources have informed the development of lessons, activities, and teaching suggestions. Children in the early grades are capable of much more than had been previously thought. Manipulatives facilitate modeling mathematical concepts and communication about those concepts, thus promoting the development of children's thinking. The problem-solving approach and everyday contexts in *Everyday Mathematics* are similar to lessons in Japanese classrooms and other constructivist classes, but are also based on Dewey's conception of inquiry-based learning that connects to students' everyday knowledge.

More than 50 years of scholarship provided an initial research base for curriculum development, but a writing process that deeply involves teachers and children in the development of the materials has been essential to translating that research into practice. Teachers using *Everyday Mathematics* have been generous in opening their classes for observations and tests and in helping us identify strengths and weakness of the existing K-6 *Everyday Mathematics* program. Through meetings, surveys, classroom observations, and interviews, these teachers continue to provide much valuable advice for the authors as they continuously work to improve the curriculum.

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